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AN EXAMINATION OF THE USAF (Q,R) POLICIES FOR MANAGING DEPOT-BA--ETC(U)
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An Examination of the United States
Air Force (Q,R) Policies for Managing
Depot-Base Inventories

by

Leroy B. Schwarz

October 15, 1976

Prepared for: Air Force Business Research Management Center,
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This paper reports the results of a computer simulation experiment which compared the operating performance of the USAF current policy for managing depot-base inventories of nonreparable spares with the operating performance of two alternative policies: a system myopic policy, and an allocation policy. Operating performance measures were: (1) average annual order plus holding cost; (2) average annual order plus acquisition cost; and (3) average annual backorder-days at the bases. The study

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Abstract

This paper reports the results of a computer simulation experiment which compared the operating performance of the USAF current policy for managing depot-base inventories of nonreparable spares with the operating performance of two alternative policies: a system myopic policy and an allocation policy. Operating performance measures were: (1) average annual order plus holding cost; (2) average annual order plus acquisition cost; and (3) average annual backorder-days at the bases. The study used demand history for a sample of 50 items currently stocked by the Ogden and Oklahoma depot systems. The report concludes that, at least under certain circumstances, system operating performance may be improved, perhaps substantially, by adoption of either of the proposed alternative policies.

I. Objective of the Study

This study examines the United States Air Force (USAF) current policy for managing depot-base inventories of nonreparable spares. This policy may be loosely described as an independent (Q,R) policy; that is, each stocking point in the system (depot or base) determines how much to order, Q, and when to order, R, using models which ignore the fact that the given stocking point is part of a depot-base distribution system. In particular, the model used to determine (Q,R) for the depot assumes that the depot supplies customer demand directly, without going through base inventory. Correspondingly, the model used to determine (Q,R) for the bases assumes that each base receives stock from a supplier outside the system, instead of receiving stock from a depot within the system.

The designers of the current USAF (Q,R) policy were well aware of these assumptions. The current policy is intentionally a suboptimal one from the standpoint of minimizing system order, holding, acquisition, and backorder performance. However, despite this suboptimality, the current USAF policy has two very desirable qualities: (1) it is very easily computed (Q's and R's are easily computed); and (2) it is very easily implemented; i.e., each stocking point may operate independently, without bothering to coordinate its policies and/or operations with other stocking points.

The purpose of the study reported here was to compare the operating performance of current Air Force policy with that of two alternative

policies in order to determine if current policy might be improved. Performance measures used were:

1. Average Annual Order and Holding Cost;
2. Average Annual Ordering and Acquisition Cost;
3. Average Annual Acquisition Cost;
4. Average Backorder-Days[1] at the Bases;

In order to measure the operating performance of the current and alternative policies, a computer simulation of a depot-base inventory system was designed and constructed. This program, which is briefly described in Section III, has the capability of simulating the day-by-day transactions for <u>any</u> set of items currently managed by the USAF (Q,R) system according to <u>any</u> given continuous review policy, including the current policy, under a variety of operating conditions. This program was used to evaluate the given policies using the demand history for a sample of 50 items currently stocked at the Ogden and Oklahoma depot systems. See Section III for details.

In Section IV we report the results of our tests. Due to the relatively small number of items examined and the nonstationarity of the system during the period of observation, our conclusions must be tentative. However, our tests do indicate that at least under certain circumstances current Air Force policy may be improved, perhaps substantially, by the adoption of either the alternative policies proposed here. See Section IV for details. Further tests, suggested in Section V, are required to substantiate these results.

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[1]A backorder-day is the equivalent of one backorder outstanding for one day.

II. Policies Tested

In this section we describe each of the 3 policies tested in the study:

1. Current Air Force Policy

2. The "System Myopic" Variant to Current Air Force Policy

3. The Allocation Policy

Details are provided in Appendices A-D.

Current Air Force Policy (CURRENT)

The current Air Force policy (henceforth called CURRENT) uses an independent (Q,R) policy for each item.

Base (Q,R)

Each base stocking the given item determines how much to order, Q, and an inventory reorder level, R, independent of the costs and/or policy of the depot supplying the item to it and independent of the costs and policies of any other bases stocking the item. Whenever the inventory position (on-hand + on-order - backorders), I, of a given item at a given base falls below its reorder point, R, a quantity Q is ordered which is equal to an adjusted[2] Wilson EOQ (Economic Order Quantity) plus the deficit, if any, between R and I. This restores the inventory position for the item to the adjusted EOQ + R. The base reorder point, R, is set equal to the given item's average demand during the nominal lead time

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[2] If Wilson EOQ exceeds one year's supply, it is adjusted downwards to one year's supply; if it is less than 30 days' supply, it is adjusted upwards to 30 days' supply.

for the depot to supply the item to the base plus a safety stock, S, as a cushion against above average demand or longer than nominal lead times. See Appendix A for details.

Depot (Q,R)

The depot stocking the given item determines its (Q,R) independent of the costs and/or policies of the bases it supplies. The Q determination is similar to that used by the bases. However, the determination of the depot safety stock is somewhat more complex, based on a model by Presutti and Trepp [2]. See Appendix A for details.

A System Myopic Variant of Current Air Force Policy (MYOPIC)

The system myopic variant of CURRENT policy (henceforth called MYOPIC) is also a simple (Q,R) policy. The reorder levels prescribed for bases and depots are identical to those of CURRENT. The difference is in the determination of the depot and base order quantities. Instead of being based on the Wilson EOQ model, the Q calculations in the MYOPIC policy are based on the system myopic heuristic of Schwarz, [1], [3] and [4]. This heuristic has been demonstrated to yield near-optimal Q values for deterministic depot-base inventory systems. Although this model is only a deterministic one (as is the Wilson EOQ model) it does provide a simple scheme for determining base and depot Q values which incorporates the interactions of costs and order quantities between the depot and its bases and among the bases themselves. The details for determining the system myopic Q values are given in Appendix C.

If the MYOPIC policy can be demonstrated to be superior to CURRENT from the standpoint of annual operating and/or acquisition costs, then

we will have found a policy which has most of the desired properties of CURRENT policy, with approximately the same backorder-day performance,[3] and lower operating and/or acquisition costs.

The Allocation Policy

The allocation policy (henceforth called ALLOCATION) uses the same depot and base order quantities and the same reorder levels as the CURRENT policy. However, whenever the depot receives a base order which, if filled, would draw on-hand inventory below the depot safety stock, the depot enters a rationing mode, and remains in this mode until depot on-hand inventory rises above its safety stock again. Any base demand received by the depot when the depot is in the rationing mode initiates a rationing calculation which is designed to fill that demand only partially, reserving some inventory in anticipation of other base demands. The quantity shipped to the given base in response to its demand depends upon the quantity demanded by that base and the anticipated quantities to be demanded by all of the other bases before the depot receives its next shipment. In this manner ALLOCATION attempts to spread the risk of a customer backorder across all bases, and consequently improve the overall backorder performance of the CURRENT policy. Any discrepancy between the order quantity demanded by a given base and the quantity shipped by the depot remains as a backorder at the depot to be filled upon receipt of the next depot order quantity. See Appendix D for details.

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[3] Since the MYOPIC R values and safety stocks are identical to those of CURRENT, the backorder-day performance of these two policies should be approximately the same.

III. Experimental Design

In order to evaluate the CURRENT, MYOPIC, and ALLOCATION policies, a computer simulation model of a one-depot-N-base distribution system was designed and constructed. For each simulated day base level demand for each item at each of up to 31 bases is read from a data set. Demands are filled if sufficient inventory is on hand; any deficiencies are backordered and the backorder-days are accumulated; reorder levels are checked, orders placed and shipped according to the given policy, and order costs are accumulated; holding costs for on-hand inventory are also accumulated. Similar calculations are performed for the depot, except backorders are not accumulated.[4] Partial shipments from the depot to the base are made if inventory and/or policy dictates. Deliveries to the bases are made in accordance with the lead (order and ship) times specified for the given base. Deliveries to the depot are made in accordance with the lead time specified for the given item. Lead times are deterministic. Every effort was made to carefully replicate the most important elements of the real system. However, for simplicity and clarity we have excluded returns and quantitatives from our model. Also, for convenience, we have modelled depot sales demands as demands occurring at a fictitious base operating at the depot with a lead time of one day.

Details of the simulation model are available on request.

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[4] This is because backorders at the depot are only significant to the extent that they lead to base level backorders, which are accumulated in the base level calculations.

In the remainder of this section we describe the procedure used to select the 50 items chosen for the simulation, the parameters of the experiment, the initialization procedure, and the procedure used to collect cost and performance data for each of the policies tested.

Sample Selection Procedure

The AFLC supplied unit acquisition cost, depot lead time, and a 16 quarter (4 year) depot demand history (unit demand and orders received) for 792 part numbers stocked at the Ogden depot and 186 part numbers stocked at the Oklahoma depot. Of these 792 (186) part numbers, 410 (64) were stocked by the one or more of the bases for which lead times and weighting factors were also supplied (See Appendix E). Of these 410 (64) part numbers, 12 (1) were excluded from the sample because of zero demand in each of the first 8 quarters (2 years). The remaining 398 (63) part numbers were then sorted on the basis of average quarterly depot demand (sales plus transfers less returns) for the first 8 quarters (2 years). Fifty part numbers were chosen from the resulting lists, 43 from the Ogden list and 7 from the Oklahoma list. This represent a sample of approximately 11% of the 398 + 63 = 461 usable part numbers supplied by the AFLC. The 43 (7) part numbers were chosen by partitioning each ranked list into groups of 9 parts each, the nine parts with the largest average quarterly demand in the first group on each list, the next nine in the second group, etc. [Note that 9 is the integer part of 63/7 and of 398/43.] Finally, the part having the median average quarterly demand in each group on each list was chosen. A list of the 50 chosen part numbers is available on request.

The Parameters of the Experiment

The following parameters were used in the simulation experiment:

Parameter	Value
Depot Order Cost	$270.16
Depot Holding Cost	20% of Acquisition Cost/Unit
Base Order Cost	$5
Base Holding Cost	50% of Acquisition Cost/Unit
Base Lead Times	See Appendix E

Unit acquisition cost and depot lead times were supplied for each item by the AFLC. In cases where more than one unit cost and/or depot lead time was supplied, the most recent value was chosen. Average requisition size for the depot safety stock calculation was calculated from the data supplied by dividing total unit demand for years 2 and 4 by the total number of orders received during these years.

Three values of the safety factor, λ, used in the depot safety stock calculation (see Appendix A) were used in the experiment: $14.19, $113.25, and $453.00. See Appendix B for details.

Daily demand for each of the 50 items at each base in each quarter of the simulation was generated using a Poisson distribution. See Appendix E for details. Base demand rates were allowed to vary between quarters in accordance with the supplied depot demand data, but were assumed to be stationary within each quarter. Two complete sets of demand history were generated and tested for each item, using two different random number seeds.

Initialization Procedure

For each of the three policies tested, the system was initialized as follows: the first two years (8 quarters) of the supplied depot demand history was used to compute the values of: (i) mean daily base demand; (ii) mean monthly depot demand; and (iii) the quarterly depot MAD required to compute the initial (Q,R) values for each item to be used by the depot and bases. See Appendix F for details. The system was then initialized by setting depot inventory for each item equal to one-half the calculated depot Q plus lead time demand (mean depot demand x depot lead time) plus an additional one month's supply. Base inventory for each item was set equal to one-half the calculated base Q plus lead time demand (mean base demand x base lead time). Initial pipeline inventories (outstanding depot and base orders) were set equal to zero.

Data Collection Procedure

Given the values of initial inventories determined above, the day-by-day transactions of the system operating under each of the 3 alternative policies (CURRENT, MYOPIC, and ALLOCATION) for each of the 3 safety factor values (3x3 = 9 policy-safety factor combinations) were then simulated using the daily demand history for the remaining two years (8 quarters) for which depot demand history was supplied.[5] The daily demand history was held fixed for all policy-safety factor combinations. Cost and backorder-days were accumulated on a quarterly basis.

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[5]Recall that the first two years of demand history were used for system initialization.

The entire process described above was then repeated using a second demand history generated using the same depot demand history and the same procedure, but a different random number seed.

Hence the simulation experiment involved the simulation of 1800 item-years of demand data (= 50 items x 3 policies x 3 safety factors x 2 years x 2 random number seeds).

The following section reports the results obtained.

IV. Results and Analysis

The procedure described in Section III yielded (i) total order cost; (ii) total holding cost; and (iii) total backorder-days by quarter for each of the policy-safety factor-random number seed combination described in section III. In this section we examine the results obtained.

Table 1 presents the results at the highest level of aggregation possible. To prepare Table 1, the operating performance measures for each policy-safety factor combination for all 50 items were summed for each of the eight quarters observed in the demand history from one random number seed and divided by 2 to yield average annual measures. These results and their averages across the two random number seeds are presented in Table 1.

Two important observations should be made in Table 1: First, observe that for any given policy-safety factor combination the differences in performance between the two demand histories (random number seeds) is quite small, particularly relative to the differences between policies for the various safety factors. Second, observe that the differences between policies is quite large in some cases, e.g., for a safety factor of \$113.25 CURRENT yields average annual holding costs of \$189,088 and backorder-days of 1,941,458 while MYOPIC yields a lower average annual holding cost, \$158,110, and lower backorder-days, 1,842,521.

The differences in operating performance between policies may be observed better in Figures 1-3. Figure 1 displays the average

Table 1: Average Annual Performance Measures

Policy	Seed	Order + Holding Cost			Order + Acquisition Cost			Acquisition Cost			Backorder-Days		
		λ			λ			λ			λ		
		$14.19	$113.25	$453.00	$14.19	$113.25	$453.00	$14.19	$113.25	$453.00	$14.19	$113.25	$453.00
CURRENT	1	109,733	188,371	256,500	1,114,704	1,269,614	1,554,388	1,068,292	1,223,472	1,503,926	2,350,359	1,958,780	1,593,023
	2	106,385	189,805	253,793	1,061,092	1,261,660	1,487,478	1,017,922	1,215,115	1,440,123	2,462,684	1,924,137	1,640,890
	Avg.	108,059	189,088	255,146	1,087,898	1,265,637	1,520,933	1,043,107	1,219,294	1,472,024	2,406,522	1,941,458	1,616,956
MYOPIC	1	80,335	158,139	219,538	931,463	1,140,370	1,327,264	889,088	1,095,160	1,281,379	2,320,650	1,836,030	1,635,690
	2	82,298	158,082	221,471	973,273	1,137,475	1,358,158	931,053	1,094,580	1,314,318	2,289,256	1,849,012	1,565,212
	Avg.	81,316	158,110	220,504	952,368	1,138,922	1,342,711	910,070	1,094,870	1,297,848	2,304,953	1,842,521	1,600,451
ALLOCATION	1	110,007	191,963	257,585	1,114,704	1,326,660	1,554,388	1,066,792	1,277,008	1,503,926	1,998,789	1,712,514	1,491,772
	2	111,285	192,568	259,123	1,120,365	1,313,941	1,549,080	1,073,955	1,264,291	1,498,620	1,978,455	1,690,636	1,473,013
	Avg.	110,646	192,266	258,354	1,117,534	1,320,300	1,551,734	1,070,374	1,270,650	1,501,273	1,988,622	1,701,576	1,482,392

annual backorder-days (averaged over both demand histories), $\overline{BOD}$, versus average annual order + holding cost, $\overline{OC}$ + $\overline{HC}$, for each of the given policies and safety factors. Data points are represented by C's = CURRENT, M's = MYOPIC, and A's = ALLOCATION. Smooth curves have been drawn through the data points for each policy in order to highlight the apparent shape of the trade-off between backorder-days and average cost for each policy. Figure 2 displays similar curves of average annual backorder-days, $\overline{BOD}$, versus average annual order plus acquisition cost, $\overline{OC}$ + $\overline{AC}$. Figure 3 displays similar curves of average annual backorder-days, $\overline{BOD}$, versus average annual acquisition cost, $\overline{AC}$.

Analysis of Figure 1

Figure 1 shows the trade-off in backorder-days versus average annual order plus holding cost for each of the three tested policies. For example, the rightmost data point for the CURRENT policy indicates the average cost ($255,146/year) and backorder-days (1,616,956) for CURRENT policy using a safety factor λ = $453. As the safety factor declines, CURRENT employs lower and lower safety stocks, yielding smaller holding costs and more backorders. Similarly for the MYOPIC and ALLOCATION policies.

Observe that both the MYOPIC and ALLOCATION policies outperform CURRENT policy, i.e., yield lower average annual order plus holding cost for the same backorder-day performance or yield lower backorder-days for the same average annual order plus holding cost. For example,

FIGURE 1

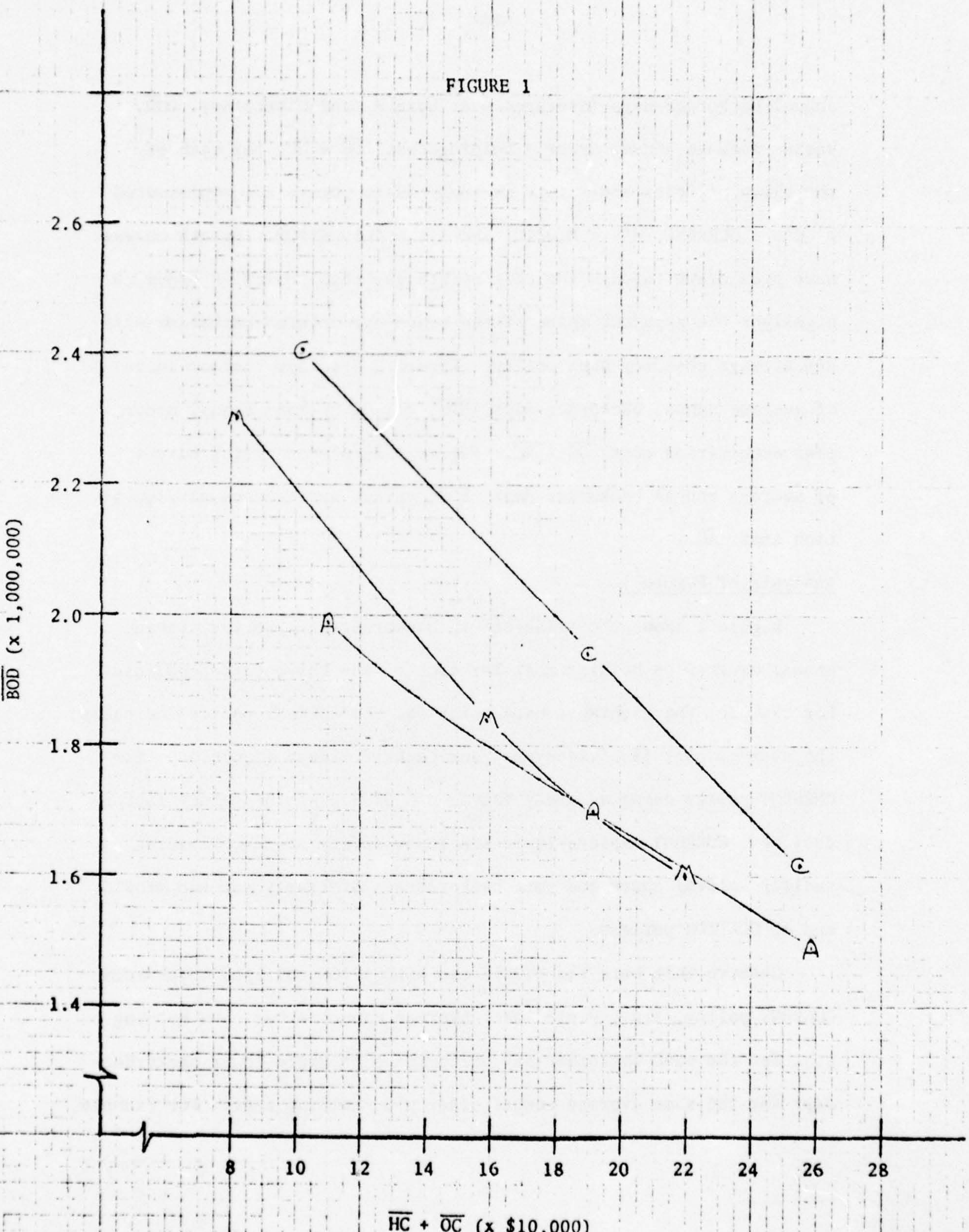

2.6
2.4
2.2
2.0
1.8
1.6
1.4
$\overline{BOD}$ (x 1,000,000)
8
10
12
14
16
18
20
22
24
26
28
$\overline{HC}$ + $\overline{OC}$ (x \$10,000)
C
M
A

FIGURE 2

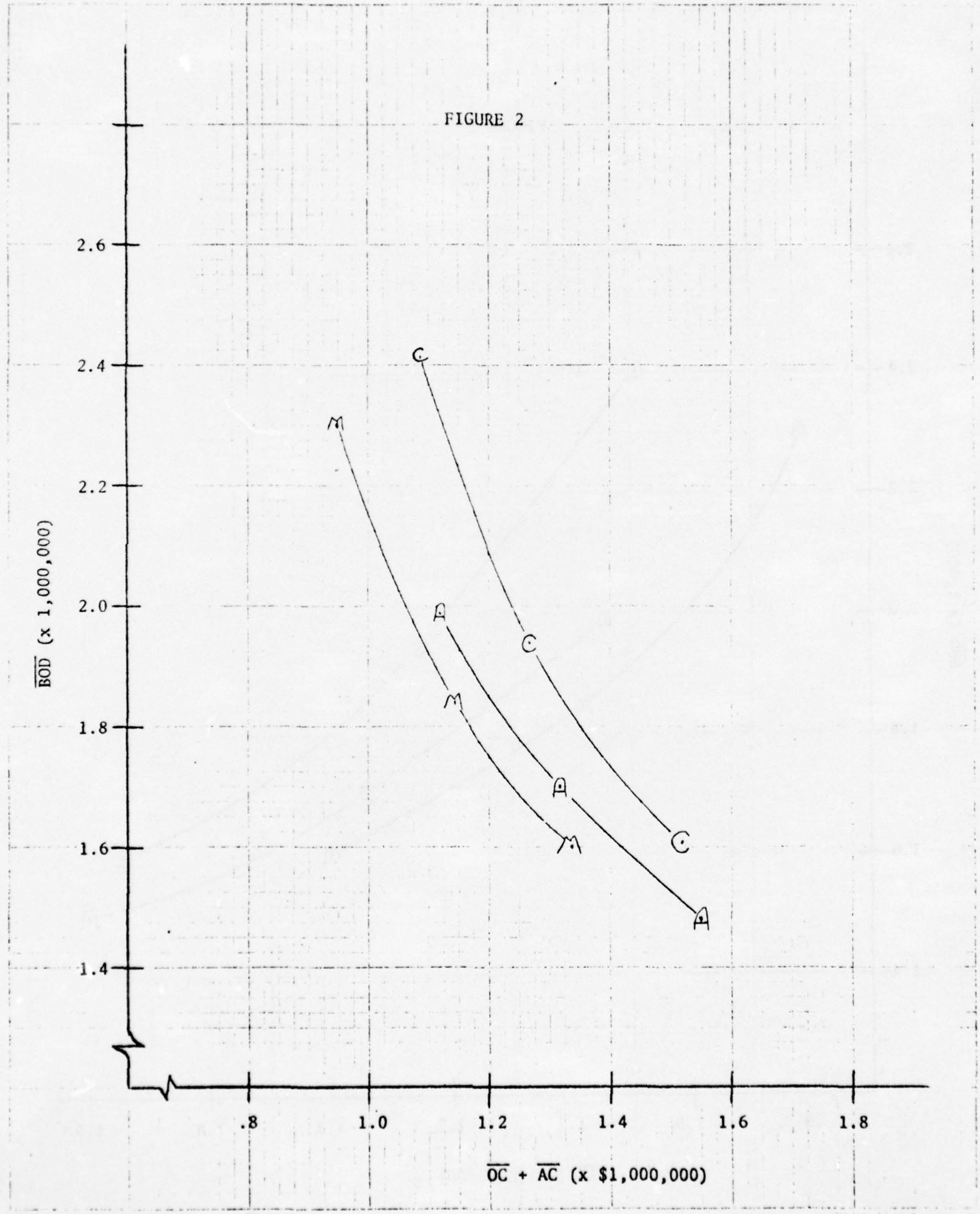
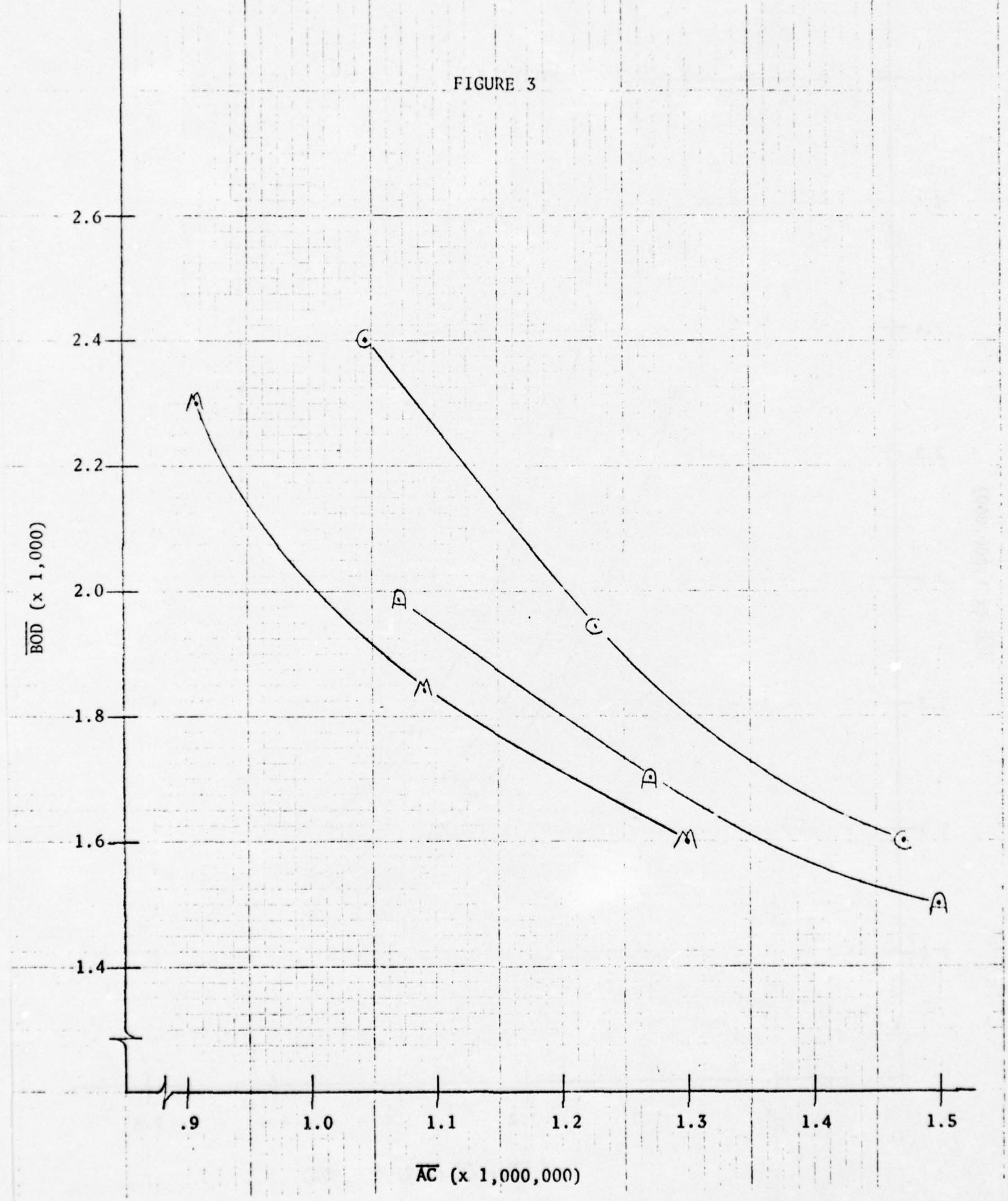

2.6
2.4
2.2
2.0
1.8
1.6
1.4
$\overline{BOD}$ (x 1,000,000)
.8
1.0
1.2
1.4
1.6
1.8
$\overline{OC}$ + $\overline{AC}$ (x \$1,000,000)
C
M
A

FIGURE 3



if each tested policy were adjusted (by altering the safety factor) to yield average annual order plus holding costs of \$160,000, Figure 1 indicates that the ALLOCATION policy would yield $\overline{BOD}$ approximately equal to 1.8 million/year; for the MYOPIC policy, $\overline{BOD}$ would equal approximately 1.83 million/year; and CURRENT policy, $\overline{BOD}$ would equal approximately 2.1 million/year. Similarly, if each policy were adjusted to yield $\overline{BOD}$ = 1.8 million/year, the MYOPIC and ALLOCATION policies would cost \$160,000 - \$170,000 per year in order plus holding cost, whereas CURRENT would cost about \$220,000 per year.

Figures 2 and 3

Figures 2 and 3 display the same superiority for the MYOPIC and ALLOCATION policies over CURRENT policy. Figure 2 shows that for the same average annual expenditure for order and acquisition cost, the ALLOCATION policy yields fewer backorder-days per year than CURRENT, and the MYOPIC policy yields still fewer backorder-days. Similarly for acquisition cost and backorder-day performance.

The implication of Figures 1-3 are quite clear: CURRENT policy is dominated by both the MYOPIC and ALLOCATION policies.

However, a closer analysis of the data suggests caution in interpreting the observed differences between policies. In particular, if instead of examining the average operating performance of each policy over the entire two year test period one examines the quarter by quarter operating performance, significant nonstationarities emerge.

Figure 4 displays the backorder-days (BOD's) for each policy on a quarter-by-quarter basis for each of the 8 quarters (quarters 9-16) in the test period, averaged over the two demand histories (random number seeds). As shown, the BOD's for all of the tested policies grew dramatically over the first year of the test period (quarters 9-12), and fluctuated (or grew slightly) during the subsequent year (quarters 13-16).

There are several possible causes for this nonstationarity, the most important of which are: (1) the nonstationarity of depot demand over the test period; and (2) the procedure used to initialize the simulation at the beginning of the test period. Further tests are required to test the relative magnitude of these (and other possible) sources of the nonstationarity.

Figure 5, a graph or quarterly order plus holding costs on a quarter-by-quarter basis over the same test period, also displays some nonstationarity, although the fluctuations are considerably smaller.

The implications of this nonstationarity are twofold: First, the typical tests used to analyze empirical data (e.g., significance tests, etc.) may <u>not</u> be applied to the data in any legitimate manner. Second, and more important, the nonstationarity implies that the system was possibly not behaving "typically" during the test period. For example, the observed relative backorder-day performance of the three tested policies might have been different had data been available

FIGURE 4

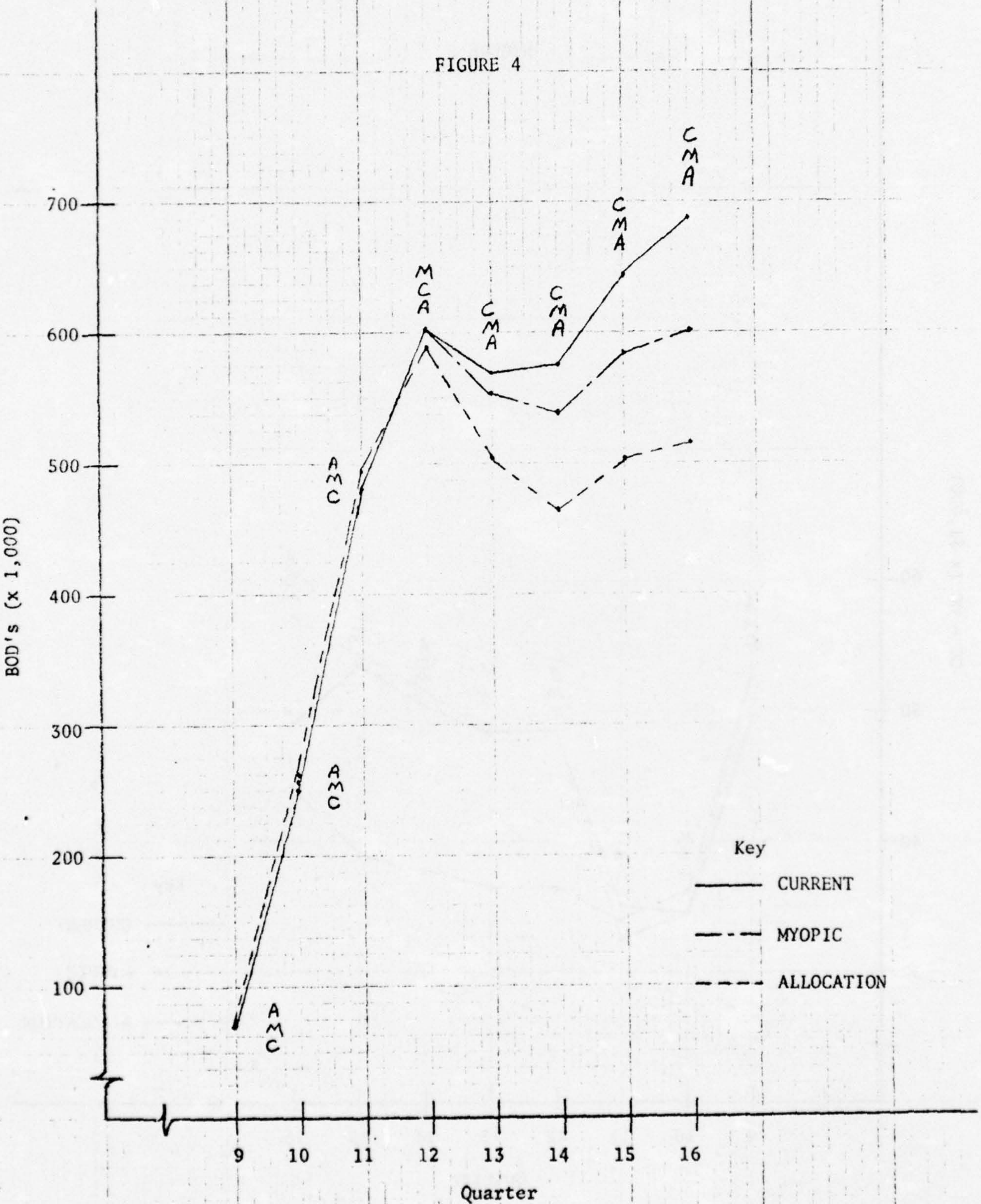
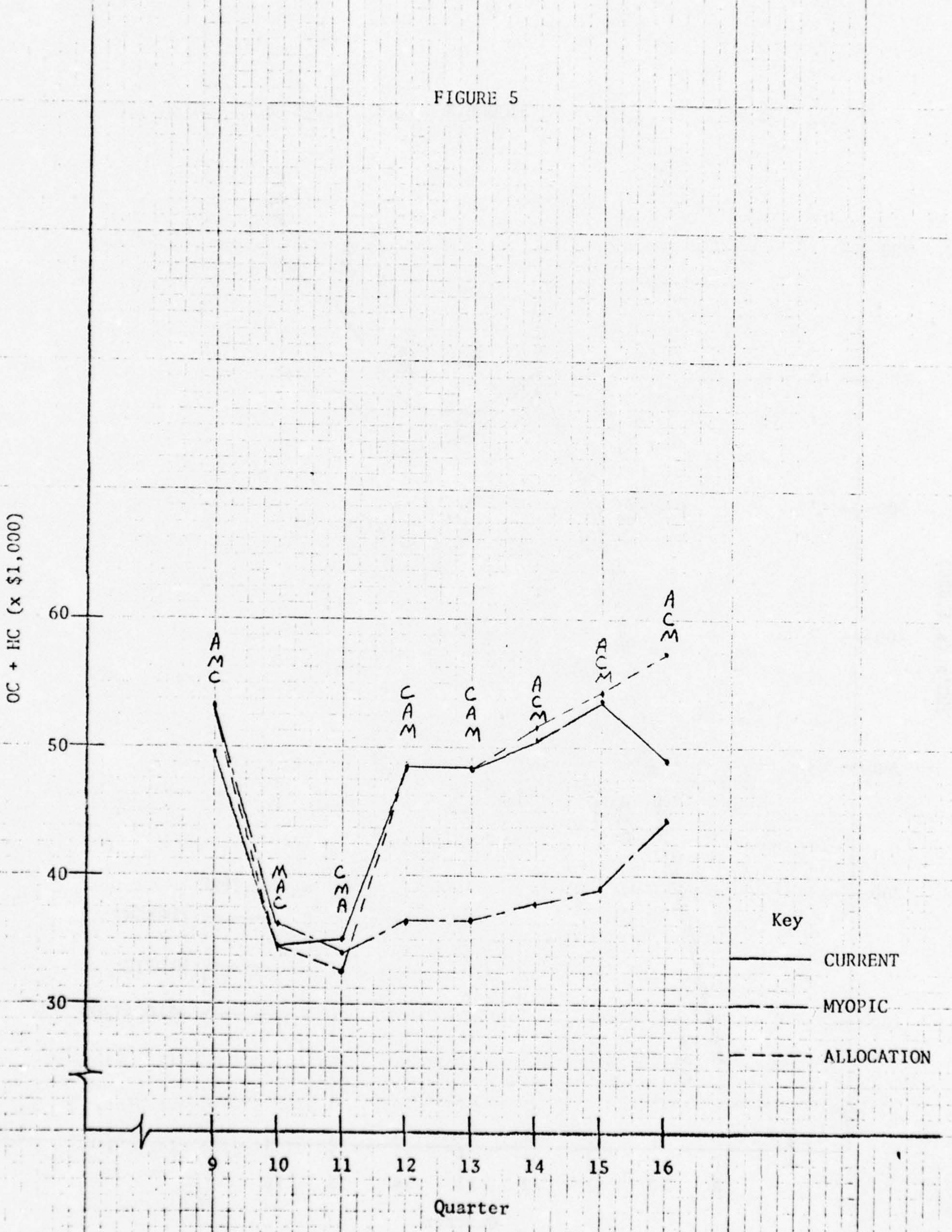

BOD's (x 1,000)
700
600
500
400
300
200
100
9
10
11
12
13
14
15
16
Quarter
Key
CURRENT
MYOPIC
ALLOCATION

FIGURE 5


OC + HC (x $1,000)
60
50
40
30
A M C
M A C
C M A
C A M
C A M
A C M
A C M
A C M
9
10
11
12
13
14
15
16
Quarter
Key
CURRENT
MYOPIC
ALLOCATION

to test the performance of the policies over a period of more than two years.

It is quite worthy of note, however, that despite the nonstationarity in the performance of the policies during the test period, the performance of CURRENT was equalled or bettered by either of the alternative policies.

Conclusion

The nonstationarity of the policies during the test period does not permit unqualified conclusions. However, if the nonstationarity is ignored, the tests described here indicate that CURRENT policy may be improved, perhaps significantly, by adoption of either the MYOPIC or the ALLOCATION policies. Furthermore, despite the nonstationarity observed during the test period, either of the alternative policies equalled or outperformed CURRENT policy.

V. Suggestions for Further Study

The research described here should be extended in three ways.

First, and most important, the cause of the nonstationarity observed in the operating performance of the tested policies should be examined. The most important questions here are the relative importance of the initialization procedure versus the nonstationarity of the demand itself. If the primary cause of the nonstationarity is the initialization procedure, that procedure should be changed and/or the simulation program allowed to run more simulated time in order to allow the effect of the initialization procedure to wear off.

If the primary cause of the nonstationarity is the nature of demand, then the three policies examined here should be reexamined under several possible nonstationarity demand situations in order to determine if the superiority of the alternative policies over CURRENT observed here is maintained under a variety of situations.

Second, the sample examined here should be extended to <u>more</u> items and, more important, to <u>extended time periods</u> (more than 4 years). This is required in order to test and measure the long term impact of the tested policies on operating performance.

Finally, other alternative policies should be tested. One such policy worthy of test is a policy that uses the Q values determined by the MYOPIC policy and the safety stock allocation procedure of the ALLOCATION policy. It is reasonable to speculate that such a policy would outperform both the ALLOCATION and MYOPIC policies, and considerably outperform CURRENT policy.

Appendix A: The CURRENT Policy

This Appendix describes the calculations of a given item's (Q,R) values for the bases and depot under the CURRENT policy.

Base (Q,R):

The formula for determining the reorder level for a given item at a given base j, R_j, is:

$$R_j = \mathrm{INT}[DDR_j \times L_j + SS_j + .5] \qquad (A1)$$

where L_j = nominal lead (order and ship) time from depot to base j, in days;

DDR_j = daily demand rate at base j (total demand at base j during the last 365 days divided by 365);

SS_j = safety stock for base j $= [3 \times DDR_j \times L_j]^{\frac{1}{2}}$;

$\mathrm{INT}[X]$ = the greatest integer $\leq X$;

The formula for determining the Q for a given item at base j, Q_j, is:

$$Q_j = \mathrm{Max}\left\{\begin{array}{l} \mathrm{INT}[30 \times DDR_j + .999]; \\ 1; \\ \mathrm{INT}[\mathrm{Min}\{365 \times DDR_j, EOQ_j\} + .999] \end{array}\right\} + (R_j - I_j) \qquad (A2)$$

where

$$EOQ_j = \left[\frac{2 \times 365 \times DDR_j \times OC_j}{HC_j \times UC}\right]^{\frac{1}{2}} \qquad (A3)$$

OC_j = base j's order processing cost = \$5;

UC = unit acquisition cost of the given item;

HC_j = cost to hold each unit of the given item/year at base j, expressed as a fraction of its unit cost

= .5;

I_j = base j's inventory position (on-hand + on-order - backorders);

The values of EOQ_j and R_j are recomputed every quarter.

Note: The safety stock calculation is based on the assumption that base lead time demand is compound Poisson with mean $DDR_j \times L_j$ and variance to mean ratio of 3. Hence SS_j is set equal to one standard deviation of lead time demand.

Depot (Q,R):

The formula for determining depot Q, Q_D, for a given item is:

$$Q_D = \text{Max} \begin{Bmatrix} \text{INT}[6 \times MDR_D + .5]; \\ 1; \\ \text{INT}[\text{Min}\{36 \times MDR_D; EOQ_D\} + .5] \end{Bmatrix} + (R_D - I_D) \qquad \text{(A4)}$$

where

MDR_D = monthly depot demand rate

$$= \frac{1}{24} \sum_{n=1}^{8} [\text{Demand in } n^{th} \text{ most recent quarter}] \qquad \text{(A5)}$$

$$EOQ_D = \left[\frac{24 \times MDR_D \times OC_D}{HC_D \times UC}\right]^{\frac{1}{2}} \qquad \text{(A6)}$$

OC_D = depot order processing cost

= \$270.61;

UC = unit acquisition cost of item;

HC_D = cost to hold each unit of the given item/year at the depot, expressed as a fraction of its unit cost

= .2

The formula for determining depot R, R_D, for a given item is:

$$R_D = \text{INT}[MDR_D \times L_D + SS_D + .5] \tag{A7}$$

where

L_D = nominal lead time from supplier to depot, in months;

SS_D = safety stock

$= \text{Max}\{K \times \sigma_D ; 0\}$;

$$\sigma_D = .5945 \times MAD_Q \times (0.82375 + 0.42625 \times L_D) \tag{A8}$$

$$K = 0.707 \times \log_e \frac{\lambda}{2 \times HC_D \times UC \times R^{\frac{1}{2}}} \cdot \frac{\sigma_D \times (1 - e^{-\sqrt{2}\, EOQ/\sigma_D})}{\sqrt{2} \times EOQ_D} \tag{A9}$$

$$MAD_Q = \frac{1}{8} \sum_{n=1}^{8} \left|\text{Demand in } n^{th} \text{ most recent quarter} - (3 \times MDR_D)\right| \tag{A10}$$

R = average requisition size

= demand during last 24 months divided by the number of orders;

λ = shortage factor.

The derivation of SS_D may be found in Presutti and Trepp [2].

The procedure used to select the values of the shortage factor, λ, to be used in the experiments is described in Appendix B.

Appendix B: Selection of λ Values

The depot safety stock calculation (see (A7)-(A9)) requires the use of a safety factor, λ. Values for λ used in the simulation experiments reported here were determined as follows:[6]

Current Air Force policy uses depot safety stock of 53 days' supply for each item. Thus, in order to select the λ value which approximates current policy we must find λ satisfying:

$$\sum_i UC_i \times SS_i(\lambda) = 53 \sum_i UC_i \times MDR_{Di}/30 \qquad \text{(B1)}$$

for the sample of items (i) considered, where

UC_i = unit acquisition cost of the i^{th} item;

$SS_i(\lambda)$ = depot safety stock for the i^{th} item determined by equation (A9);

MDR_{Di} = average monthly depot demand rate for item i.

The value of λ which came closest to satisfying (B1) for the sampled items was $113.25.

The other λ values, λ = $14.19 and λ = $453, were chosen by trial and error to yield increases and decreases in average annual order plus holding costs of approximately 25%.

The choice of all of these λ values is, of course, arbitrary. However, inasmuch as our interest concerns only the relative performance of the three alternative policies over a range of λ values, the choice of specific λ values is irrelevant provided that the Air Force's "true" safety factor lies in the chosen range of between $14.19 and $453 per backorder-day.

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[6]This procedure was suggested by Mr. Victor Presutti, AFLC.

Appendix C: The MYOPIC Policy

This Appendix describes the calculation of a given item's (Q,R) values for the bases and depot under the MYOPIC policy.

R Determination:

Identical to CURRENT

Q Determination:

The Q values for depots and bases is based on the system myopic heuristic of Schwarz. See below.

For the depot Q_D satisfies

$$Q_D = \text{Max} \left\{ \begin{array}{l} \text{INT}[6 \times MDR_D + .5]; \\ 1; \\ \text{INT}[\text{Min}\{36 \times MDR_D; Q_D'\} + .5] \end{array} \right\} + (R_D - I_D) \qquad \text{(C1)}$$

where

MDR_D = monthly depot demand rate (see (A5));

$$Q_D' = \left[\frac{2 \times 12 \times MDR_D \times (OC_D + OC_B \sum_{j=1}^{N} n_j)}{UC \times (HC_D + (HC_B - HC_D) \times \sum_{j=1}^{N} (n_j MDR_j / MDR_D))} \right]^{1/2} \qquad \text{(C2)}$$

where

OC_D = depot order processing cost

= \$270.16;

OC_B = base order processing cost

= \$5;

UC = unit acquisition cost of item;

HC_D = cost to hold each unit of the given item/year at the depot, expressed as a fraction of its unit cost

= .2;

HC_B = cost to hold each unit of the given item/year at each base, expressed as a fraction of its unit cost;

= .5;

MDR_j = average monthly demand rate at base j

= $30 \times DDR_j$;

MDR_D = average annual demand rate at the depot (see (A5));

N = number of bases;

The n_j value for base j is the smallest integer n satisfying

$$n(n+1) \geq \frac{OC_D \times (HC_B - HC_D) \times MDR_j}{OC_B \times HC_D \times MDR_D} \qquad \text{(C3)}$$

The Q value for base j, Q_j, satisfies:

$$Q_j = \text{Max} \begin{Bmatrix} \text{INT}[30 \times DDR_j + .999] \; ; \\ \\ \text{INT}[\text{Min}\{365 \times DDR_j ; Q_j'\} + .999] \end{Bmatrix} \qquad \text{(C4)}$$

where

$$Q_j' = \frac{Q_D \times MDR_j}{n_j \times MDR_D} \quad . \qquad \text{(C5)}$$

Note that the Q value determination is identical to that of CURRENT except that Q_D', expression (C2), replaces EOQ_D, expression (A6), in (A4) and Q_j', expression (C5), replaces EOQ_j, expression (A3), in (A2). The values of Q_D' and Q_j' are based on the system myopic heuristic for solving stationary deterministic one-warehouse (depot) N-retailer (base) one-product problems. The heuristic is derived as follows.

Consider a one-depot one-base inventory model in which it is assumed that the base must meet a known, uniform demand rate D from customers without backlogging. The base receives all of its supply from the depot which, in turn, receives its supply from a source outside of the system. The depot (base) incurs a fixed cost K_D (K_B) each time it places an order and incurs a holding cost h_D (h_B) on each unit of echelon inventory held per year. The echelon inventory of the depot is defined to be the inventory on-hand at the depot, in transit to the base, or at the base; that is, total system inventory. The echelon inventory of the base is defined to be the inventory on-hand at the base. According to these definitions, $h_D = .2 \times UC$ and $h_B = .3 \times UC$ in CURRENT Air Force policy, where UC is the unit acquisition cost of the item.

For convenience we shall assume below that delivery to the depot and between the depot and base are instantaneous. This is the same assumption made in the basic EOQ model upon which expressions (A3) and (A6) of CURRENT policy are based.

Under these assumptions the policy which minimizes average annual order plus holding cost solves:

$$\text{Min } C(Q_D, n, Q_B) = \left(\frac{K_B D}{Q_B} + \frac{h_B Q_B}{2}\right) + \left(\frac{K_D D}{Q_D} + \frac{h_D Q_D}{2}\right) \tag{C6}$$

$$\text{s.t.} \quad Q_D = nQ_B \tag{C7}$$

$$n \geq 1, \text{ integer} \tag{C8}$$

where Q_D (Q_B) is the lot size employed by the depot (base). Note that (C7) requires that the depot order quantity be sufficient to meet exactly an integer n number of base requests. Otherwise depot holding costs would be: (i) unnecessarily large; and (ii) misspecified by (C6). In (C6) each expression in parentheses represents the average annual cost of either the depot or the base operating as in the Wilson EOQ model. Note, however, that the holding cost rates are the echelon rates, not the conventional holding cost rates used in CURRENT policy's EOQ calculation.

If (C7) is used to substitute $Q_B = Q_D/n$ in (C6) and the result optimized with respect to Q_D for fixed n, one obtains

$$Q_D^*(n) = \left[\frac{2D(K_D + nK_B)}{h_D + h_B/n}\right]^{1/2} \tag{C9}$$

Note the correspondence between (C9) and the simple EOQ formula. If (C9) is substituted back into (C6) and optimized with respect to n, the optimal n, n*, may be found to be the smallest n satisfying:

$$n(n+1) \geq \frac{K_D h_B}{K_B h_D} \quad . \tag{C10}$$

Once n* is determined, Q_D^* may be determined as in (C9) and Q_B^* determined using (C7); namely $Q_B^* = Q_D^*/n^*$.

The above model determines the optimal policy for a one-depot one-base model. The corresponding one-depot N-base model is considerably

more difficult to optimize. See [1] for details. However, the model developed above may be used to obtain heuristic policies which are quite close to optimal. To see this, consider a one-depot N-base model, where D_j is the known stationary demand rate at base j; K_j is base j's fixed order cost; h_j is base j's echelon holding cost; and n_j is the number of lots shipped by the depot to base j from Q_D. Under suitable assumptions (zero initial inventory everywhere initially), the optimal stationary policy may be shown to solve:

$$\text{Min } C(Q_D, n_1, Q_1, \ldots, n_N, Q_N) = \left(\frac{K_D D_D}{Q_D} + \frac{h_D Q_D}{2}\right) + \sum_{j=1}^{N} \left(\frac{K_j D_j}{Q_j} + \frac{h_j Q_j}{2}\right) \qquad \text{(C11)}$$

$$\text{s.t.} \quad Q_D/D_D = n_j Q_j/D_j \qquad j = 1, \ldots, N \qquad \text{(C12)}$$

$$n_j \geq 1, \text{ integer} \qquad \text{(C13)}$$

where D_D is the total demand rate satisfied by the depot, $D_D = D_1 + \ldots + D_N$. Note that (C11) - (C13) is a direct extension of (C6) - (C8) to N retailers. As above, the optimal value of Q_D for fixed n_j, j = 1,...,N may be found to satisfy:

$$Q_D^*(n_1, \ldots, n_N) = \left[\frac{2D_D(K_D + \sum_{j=1}^{N} n_j K_j)}{h_D + \sum_{j=1}^{N} \frac{h_j D_j}{n_j D_N}}\right]^{1/2} \qquad \text{(C14)}$$

Note that (C14) is a direct counterpart to (C9).

The optimal n_j, n_j^*, may be found by a branch-and-bound search procedure. See [1]. However, near-optimal n values may be found by a heuristic which determines n_j by viewing each base j as if it were operating with the depot as a one-depot one-base system. Hence the term "system myopia", or system near-sightedness. Under this heuristic, the n_j is determined to be the smallest n_j satisfying

$$n_j(n_j + 1) \geq \frac{K_B(D_j/D_D)h_j}{K_j h_B} \quad . \tag{C15}$$

Note that this is identical to (C10) except the depot order cost K_B is prorated to each of the N myopic systems (base j and the depot) in proportion to that system's share of total demand, D_j/D_D.

Using (C15) the system myopic n_j are determined. Given the n_j values, Q_D is determined as in (C14). Given Q_D, (C12) may be used to determine the Q_j values.

Expressions (C2), (C3), and (C5) correspond directly to expressions (C14), (C15), and (C12), respectively, developed above.

Further analysis of system myopic policies may be found in references [1], [3], and [4].

Appendix D: The ALLOCATION Policy

This Appendix describes the calculations of the ALLOCATION policy. As described in Section II, the ALLOCATION policy uses the same depot and base order quantities and the same reorder levels as the CURRENT policy. However, whenever the depot receives a base order which, if filled, would draw depot on-hand inventory below the depot's safety stock, the depot enters a rationing mode, and remains in this mode until depot on-hand inventory rises above the safety stock again. Any base demand received by the depot when it is in the rationing mode initiates the rationing calculation described below. Let

k = index of the base initiating the rationing calculation;

N = the number of bases served by the depot;

Q_j = the most recent order quantity demanded by base j;

D_j = the date of base j's last demand;

$DATE_D$ = the anticipated date when the next shipment will be received by the depot;

DDR_j = the average daily demand rate served by base j;

From these quantities the depot calculates

$CYCL_j = Q_j/DDR_j$

= the anticipated number of days between base j's last order and base j's next order; and

$DATE_j$ = the anticipated date of base j's next order

$= D_j + CYCL_j$

From these quantities the shipment quantity to base k, S_k, is computed to be:

$$S_k = \text{Min} \left\{ \begin{array}{l} Q_k \; ; \\ \text{INT} \left[OH_D \times \dfrac{W_k \times Q_k}{\sum\limits_{j=1}^{N} W_k \times Q_k} \right] \end{array} \right\} \tag{D1}$$

where

OH_D = depot on-hand inventory; and

$$W_j = \text{Max}\{0; DATE_D - DATE_j\} \tag{D2}$$

The discrepancy between base k's order quantity, Q_k, and the amount shipped by the depot to base k, S_k; that is, $[Q_k - S_k]$, will be automatically shipped to base k when depot inventories permit. In our cost calculations it is assumed that these extra shipments incur no extra cost. However, such additional costs may be easily included in subsequent experiments.

Appendix E: Base Level Demand Generation Procedure

Base level demand for a given item was generated for the simulation using a Poisson distribution with the mean daily demand rate for base j in quarter t, D_{jt}, computed as follows:

$$D_{jt} = \frac{F_j}{\sum_{i=1}^{N} F_i} \cdot M_t \qquad t = 1,\ldots,8 \qquad \text{(E1)}$$

where

F_j = Weighting factor for base j (see table below);

N = Number of bases handling item;

M_t = Depot daily demand rate in quarter t

= observed depot demand in quarter t divided by 90.

The following table lists the base code; base weighting factor, F_j, and base lead times for the bases used in this study.

Table E1

Base Code	F_j	Lead Time, L_j
FB2647	0.3	19 days
FB2823	3.0	11 days
FB4801	9.6	9 days
FB4802	9.6	9 days
FB4803	7.6	12 days
FB4809	7.2	10 days
FB4812	9.7	11 days
FB4829	6.6	10 days
FB4852	3.7	9 days
FB4857	3.6	12 days
FB4814	8.9	13 days
FB4887	0.6	8 days
FB5000	2.4	17 days
FB5210	5.4	28 days
FB5219	4.8	28 days
FB5250	3.0	25 days
FB5264	1.8	25 days
FB5270	9.0	16 days
FB5284	3.6	17 days
FB5294	1.8	20 days
FB5529	3.6	22 days
FB5573	7.2	18 days
FB5587	3.6	32 days
FB5606	6.6	22 days
FB5612	3.6	22 days
FB5620	4.2	18 days
FB5621	4.8	18 days
FB5643	5.4	19 days
FB5644	4.8	19 days
FB5688	1.8	14 days

Appendix F: Initial Values of DDR_j, MDR_D, and MAD_Q

Initial values of the mean daily demand rate for each item at base j, DDR_j; mean monthly depot demand, MDR_D; and quarterly depot MAD, MAD_Q were computed as follows:

$$DDR_j = \sum_{t=1}^{8} D_{jt}/720 \tag{F1}$$

$$MDR_D = \sum_{t=1}^{8} D_t/24 \tag{F2}$$

$$MAD_Q = \sum_{t=1}^{8} |M_t - MDR_D|/8 \tag{F3}$$

where D_{jt} is defined as in (E1) and D_t is the observed depot demand in quarter t.

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